STATISTICS (8-10 Questions)

Study of Data in Details.

=> Study of AM, GM, HM, Mode, Median, etc.

Q-1) The census of India is
(a) Primary Data (Datum -> singular of Data.)
(b) Secondary Data.

Note:-> Primary Data mean first hand informations where collector of data can be analyser too.

Secondary data mean informations transferred from one hand to other hand for their analysis.
Reports from Magazines & Newspapers.

Types of Data on the basis of modes of procurement:-

1) Individual Series of Data.
Eg.-> Marks obtained by 5 students as 60, 70, 80, 85 & 95.
ie. No observation is repeated

2) Discrete Series ~~Series~~ of Data / repeating Data:-
eg.-> Marks obtained by 5 students.
ie. 60, 60, 75, 82 & 99.
ie. ~~Atleast~~ Atleast one info must be repeating.

3) Grouped Data :- Continuous / Discontinuous.

Marks obtained (L – U)	No. of Students.
0–10	5
10–20	10
20–30	12
30–40	6
40–50	8

IMPORTANT TERMS.

(1) RANGE of any data is the difference b/w largest value (L) and smallest value (S).

$\therefore$ RANGE = L − S

Coefficient of Range = $\frac{L-S}{L+S}$.

e.g.→ find Range of 30, 40, 50, −2, 3, 7, −6 & 10

L = 50

S = −6

Range ⇒ L − S ⇒ 50 − (−6) ⇒ 56.

(2) Class mark/Midvalue of a Class (m) :→ $\frac{U+L}{2}$.

where, U = Upper limit

L = Lower limit.

3) Class width / class interval / class size (i) :-

$$\boxed{i = U - L}$$

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Q:-> For a class, its mark is 48.
its size is 16.
find (i) its Lower limit (LL)
(ii) its UL.

$m = 48 = \frac{U+L}{2}$

~~48~~

$i = 16 = U - L.$

$U + L = 96$
$U - L = 16$

$U = \frac{112}{2}$

$\boxed{U = 56}$

M.T.:->

(i) $LL = m - \frac{1}{2} i = 40.$

(ii) $UL = m + \frac{1}{2} i = 56.$

Q:-> In a frequency Distribution of 100 classes, lower limit of the first class is 8 & class width of each class is 10. Find (i) lower limit of 85th class
(ii) Upper " " 95th class.

L	U
8	18
18	28
28	38

100th class.

∵ class size is 10.
use AP formula.
$a_n = a_{85} = 8 + 84(10)$
Lower limit => 848.
Similarly for UL,
$a_{95} = 18 + 94(10)$
=> 958.

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ARITHMETIC MEAN (A.M.)

It is denoted by $\bar{x}$.

(1) For individuals series,

Say $x_1, x_2, x_3, \dots x_n$

$$\bar{x} = \frac{\text{Sum of all obs.}}{\text{Total no. of obs.}}$$

$$= \frac{x_1 + x_2 + \dots + x_n}{n}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$\therefore \boxed{\Sigma x = n\bar{x}}$$

(2) For Discrete Series.

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{f_1 + f_2 + \dots + f_n}$$

$$= \frac{\Sigma f(x)}{\Sigma f}$$

where f_1 = frequency of x_1

f_2 = frequency of x_2

& so on...

For grouped Data;

$$\bar{x} = \frac{m_1 f_1 + m_2 f_2 + m_3 f_3 + \dots + m_n f_n}{f_1 + f_2 + \dots + f_n}$$

$$= \frac{\Sigma(mf)}{\Sigma f}$$

Where m_1 = Midvalue of class with freq. f_1
m_2 = " " " " " f_2
& so on.

4) **Short-cut Method / Method of Assumed Mean:→**

$$\boxed{\bar{x} = A + \frac{\Sigma d}{n}}$$

Where A = Assumed Mean
$d = x - A$.
n = No. of observations.

5) Weighted Mean (W.M.) :—

$$W.M. = \bar{x} = \frac{w_1x_1 + w_2x_2 + \ldots\ldots}{w_1 + w_2 + \ldots\ldots}$$

$$\boxed{W.M. = \frac{\Sigma w_n x_n}{\Sigma w_n}}$$

Where, w_1 = weight of/weightage of x_1
w_2 = " " of x_2 & so on...

6) Combined Mean / Mean of Means / Overall Mean:→

$$\boxed{\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + \ldots\ldots}{n_1 + n_2 + \ldots\ldots}}$$

Where, $\bar{x}_1$ = Mean of n_1 obs.
$\bar{x}_2$ = " of n_2 "
$\bar{x}$ = Mean of above Means.

PROBLEMS OF STATISTICS.

Q.) Find Mean of first fifty natural numbers :-

(a) 25

~~(b)~~ 25.5

(c) 26

(d) None.

$\frac{50\times51}{2} \Rightarrow$ Sum

mean $\Rightarrow \frac{\frac{50\times51}{2}}{50}$

Note:→ Here series is 1, 2, 3, ... 50.

learn :- (Binomial Expansion)

1) $\Sigma n = 1+2+3+\dots+n$

$= \frac{1}{2} n(n+1)$

2) $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$.

3) $\Sigma n^3 = 1^3+2^3+\dots+n^3$.

$\Rightarrow \left(\frac{n(n+1)}{2}\right)^2$.

4) $2+4+6+\dots+2n = n(n+1)$.

5) $1+3+5+\dots+(2n-1) = n^2$.

Q.) Find Mean of .....

4, 9, 16, ..., 400.

Sum $2^2+3^2+4^2+\dots+20^2$.

$\Rightarrow 1^2+2^2+3^2+4^2+\dots+20^2-1^2$.

$\Rightarrow \frac{20\times21\times41}{6}-1$

$\Rightarrow 2869$

No. of obs. = 19

$\bar{x} \Rightarrow \frac{2869}{19}$ 151

Q-> Find mean of
8, 27, 64, ... 1000.

~~ans => 1000~~

$\Rightarrow 2^3 + 3^3 + 4^3 + \dots 10^3$

$\Rightarrow 1^3 + 2^3 + 3^3 + \dots + 10^3 - 1^3$

216
343
512
729
1000

$\Rightarrow \frac{1}{2}(10 \times 11)^2 - 1 \qquad \left(\frac{n(n+1)}{2}\right)^2$

$\Rightarrow (55)^2 - 1$

$\Rightarrow 56 \times 54$

3024 ÷ 9
336

No. of obs. => 9.

$\left(\frac{10 \times 11}{2}\right)^2 - 1$

$\therefore \bar{x} = \frac{56 \times \cancel{54}^{6}}{\cancel{9}_{1}} \Rightarrow 336.$

5.

Q-> Find mean heart size:-

Heart diameter (cm)	No. of students	fx
2	5	10
4	15	60
6	6	36
8	20	160
10	14	140
	Σf = 60.	Σfx => 406

$\bar{x} \Rightarrow \frac{406}{60} \Rightarrow 6.8$ cm.

Make this table.

Height in cm	No. of Students	$m = \frac{u+l}{2}$	mf
0 - 2	5	1	5
2 - 4	15	3	45
4 - 6	6	5	30
6 - 8	20	7	140
8 - 10	14	9	126
			$\Sigma fm = 346$

$$\bar{x} = \frac{346}{60} = 5.76 \text{ cm.}$$

Imp.

Q-7

$d = (x-A)$

To find mean of 100 obs. by a ~~simpler~~ simpler method, a student substracts 50 from each observation such that sum of these differences is found to be -50.

Find Actual mean.

$n = 100$

$$\bar{x} = A + \frac{\Sigma d}{n}.$$

Here, $A = 50$

$\Sigma d = -50.$

$n = 100.$

$$\boxed{\bar{x} = 49.5}$$

Q-) Use method of assumed mean to find mean of first five natural numbers.

x	A	$d = x - A$
1	↑	-1
2		0
3	2	1
4		2
5	↓	3
		$\Sigma d = 5$

lets, Assume mean = 2.

$$\therefore \bar{x} = A + \frac{\Sigma d}{n}.$$

$$\Rightarrow 2 + \frac{5}{5}$$

$$\Rightarrow \underline{\underline{3}}.$$

But in these kind of question we,

$$\bar{x} = \frac{n+1}{2}$$

$$\Rightarrow \frac{5+1}{2}$$

$$\boxed{\bar{x} \Rightarrow \underline{3}}$$

Imp.

30 is assumed Mean.

Q.) The sum of deviations when measured from 30 is -20

& " " " " " " " from 50 is 60

-50 is Assumed mean.

Find Sample size(n) & actual Mean.

$\bar{x} = A + \frac{\Sigma d}{n}$

$\bar{x} = 30 + \frac{(-20)}{n}$

$\bar{x} = 50 + \left(\frac{60}{n}\right)$

$\frac{+20}{n} = 20 + \frac{60}{n}$

$+20 = 20n + 60$

~~20n = 40~~

$n = \cancel{-2}$ $n = 1$

Q.) The sum of deviations of all obs. from actual mean is always :-

(a) 0

(b) 1

(c) -1

(d) None.

$\Rightarrow (\bar{x} - x_1) + (\bar{x} - x_2) + (\bar{x} - x_3) + \ldots + (\bar{x} - x_n)$

$\Rightarrow n\bar{x} - (x_1 + x_2 + x_3 + \ldots + x_n)$

$\Rightarrow 0$

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Q-> Find weighted mean of first thirteen naturals whose weights are numbers themselves.

$$W.M. = \frac{1\times1 + 2\times2 + 3\times3 + \dots + 13\times13}{1+2+3+\dots+13}$$

$$\Rightarrow \frac{1^2+2^2+3^2+\dots+13^2}{1+2+3+\dots+13} = \frac{\text{Formula forsa.}}{\text{" " Mansaalmo}}$$

$$\Rightarrow \frac{\frac{13\times14\times27}{6}}{\frac{13\times14}{2}} \Rightarrow 9.$$

M.T. ⇒ In such case:-

$$W.M. = \frac{2n+1}{3}$$

Q-> Find overall mean marks

Class	Mean Marks	No. of studs...
X	50	40
XII	70	60.

Formula No. 6 Combined mean

$$C.M. \Rightarrow \frac{50\times40 + 70\times60}{40+60}$$

$$\Rightarrow \frac{2000+4200}{100} \Rightarrow 62.$$

Q→ Mean of 100 obs. is found to be 30, but later on it is detected that reading of 28 & 30 were wrongly taken as 25 & 32. Find ~~actual~~ mean now actual

$\bar{x} = 30$

$n = 100$. (obs. → observations.)

Sum of $\boxed{100}$ obs. $= 30 \times 100 \Rightarrow 3000$.

$\Sigma x = n\bar{x}$ (Formula No-1.)

$\therefore$ Correct Sum $=$ 3000 − (Sum of wrong value) + (Sum of correct values)

$\Rightarrow 3000 - (25+32) + (28+30)$

$\Rightarrow 3001$

Actual Mean $= \frac{3001}{100} \Rightarrow 30.01$

Q→ In 10 innings, mean runs of a player is 75. Including 11th inning, mean run is 79.

Find Runs in 11th innings.

$\Sigma x = n\bar{x}$

$\Sigma x = 10 \times 75 \Rightarrow 750$

In 11 innings Mean(Σx) $= 79 \times 11 \Rightarrow 869$.

$\therefore$ Req. Runs $\Rightarrow 869 - 750 \Rightarrow 119$ runs.

Conceptual.

8P theory

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Q-) Mean of $\boxed{100}$ obs. is 20. When each obs. is defined by x, find mean of another $\boxed{100}$ obs. when each obs. is defined by V.

Given $\boxed{2V = 3x + 10}$

$\bar{x} = 20$ $n = 100$.	$n = 100$
$\Sigma x = 2000$	Total $\Sigma V = V \times 100$.

$$2V = 3x + 10$$

$$V = \frac{3x}{2} + 5.$$

Each observation is $\times \frac{3}{2}$ & increased by 5 Hence,

$$\boxed{\therefore \bar{V} = \frac{3\bar{x}}{2} + 5}$$

$$= \frac{3(20)}{2} + 5$$

$$\Rightarrow 35$$

Important Rule:-> If each obs. is inc./dec./multiplied/divided by a fixed Quantity new mean changes accordingly by the same quantity.

Eg. -> Mean of $x_1\ x_2\ x_3 \ldots x_n$ is $\bar{x}$.

Q-) If each is dec. by $\bar{x}$ new mean is $\underline{\underline{0}}$.

i) decreased by say a random number 6 then the mean is decreased by 6. $(\bar{x} - 6) \rightarrow$ New Mean.

Geometric Mean (G.M.)

(1) G.M. b/w a & b = $\sqrt{ab}$.

2) G.M. b/w a, b & c = $(abc)^{1/3}$

3) G.M. b/w a, b, c & d = $(abcd)^{1/4}$.

4) G.M. b/w $x_1, x_2, x_3 \ldots x_n$ = $(x_1 x_2 x_3 \ldots x_n)^{1/n}$

= (product of n observation)$^{1/n}$.

Harmonic Mean (H.M.) ⇒

(1) H.M. b/w a & b = $\dfrac{2}{\left(\frac{1}{a}+\frac{1}{b}\right)}$ = $\dfrac{2ab}{a+b}$.

(2) H.M. b/w a, b & c = $\dfrac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

(3) H.M. b/w a, b, c & d = $\dfrac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$.

(4) H.M. b/w ~~a, b, c, d,~~ $x_1, x_2, x_3, x_4, \ldots x_n$ is

⇒ $\dfrac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\ldots+\frac{1}{x_n}}$.

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Harmonic Mean = $\dfrac{\text{No. of observations}}{\text{sum of their reciprocals}}$

Relations :-

(1) A.M. > G.M. > H.M.

ie. AM is the greatest

HM is the ~~least~~ smallest.

(2) $A.M. \times H.M. = (G.M.)^2$

(3) If $a = b$;

then AM = HM = GM.

eg. ⇒ $a = 4$ $b = 16$

AM = 10

GM = 8

HM = 6.4

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Little bit Important — Avoid to use these

T.T. → ① If speeds are different but length of journey is the same then average speed = H.M. b/w speeds.

② If speeds are different but time period of journey in each case is the same then av. speed = A.M. b/w speeds

(b/w → between.) (av. → average.)

Q-1 A car covers each one-third of its journey at speeds 60, 80, 120 kmph.

Avg. speed = ??

$$t \Rightarrow \frac{\frac{x}{3}}{60} + \frac{\frac{x}{3}}{80} + \frac{\frac{x}{3}}{120}$$

$$\Rightarrow \frac{\frac{4x+3x+2x}{3}}{240} \Rightarrow \frac{3x}{240}$$

$$S \Rightarrow \frac{x}{3x} \times 240 \quad 80 \text{ kmph.}$$

Q→ Find GM b/w 2, 4, 8, 16 & 32.

Product of Sobs. $= 2 \times 2^2 \times 2^3 \times 2^4 \times 2^5$

$= 2^{1+2+3+4+5}$

$\Rightarrow 2^{5 \times 6/2} \Rightarrow 2^{15}$ GM $\Rightarrow 8$.

N.T. ⇒ If a, b, c, d & e are in G.P. such that $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = \frac{e}{d}$.

then G.M. = c.

Q→ Find G.M. b/w 1, 2, 4, 8, 16 ..... 2^n.

Product ⇒ $1 \times 2 \times 4 \ldots\ldots\ldots$

$= 2^{n(n+1)/2}$

No. of terms / observation ⇒ $n+1$.

$\therefore$ GM $= (\text{product})^{1/n+1} = 2^{n/2}$

Q→ Which is correct seq. of AM, GM & HM resp.

(b) 4, 2 & 1 ✓

~~AM ≥ GM~~

Scanned by CamScanner

Q→ H.M. b/w two numbers is 6.4
G.M. " " " is 8.

Find 2 no.s.

$\frac{2ab}{a+b} = 6.4$
$\sqrt{ab} = 8$
} Don't solve check option.

$\frac{2 \cdot 64}{a+b} = 6.4$

$a+b = 20$

16 & 4 ✓

MODE :-

Q:-) Is mode an average?
(a) Yes (b) No.

Note:-> Mode is a term derived from 'lamode' a french word means fashion.
∴ Mode is called fashionable average / ~~quantitative~~ qualitative average.

Mode = Most frequent Observation.
→ ~~No~~ The observation with max. frequency.

(#) For grouped data, Mode $= L + \left(\dfrac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times i$

where L = lower limit of Modal Class i.e. Class with max. freq.
f_0 => freq. of modal class
f_1 => freq. of pre-modal class.
f_2 => freq. of post-modal class.
i => Class size

Steps to keep in Mind.

→ Choose interval with max. freq. this is our Modal class.

→ Select L, f_0, f_1, f_2 & i.

f_0 → freq. of modal class
f_1 → freq. of pre modal class.
f_2 → freq. of post modal class.

Q-1) Find Mode for first fifty natural no.s

(a) 25

(b) 25.5

(c) 26

✓(d) None.

Note:→ here series is 1, 2, 3 50

∴ None of them is repeated

∴ Mode doesnot ~~repeat~~ exist.

Q-2) Find Modal heart size

Size heart Diameter (cm)	No. of persons.
0-2	3
2-4	8
4-6	12
$\boxed{6-8}$	$\boxed{13}$
8-10	2

Modal class.

$$\text{Mode} \Rightarrow 6 + \left(\frac{13-12}{26-12-2}\right) \times 2 \Rightarrow 6 + \frac{1}{12} \times 2$$

$$\Rightarrow \frac{73}{12} \times 2 \Rightarrow 6.2\text{[illegible] cm}$$

To check.

Lower Modal class < Mode < Higher Modal class.

MEDIAN

Q→ Is median an average?
(a) Yes (b) No

Note :→ Median is also called positional average/ Quantitative average.

Where all observations are arranged in inc/dec order,

Median = Middle most observation.
& it is found using rules.

(1) If no. of observation = n (odd)

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{obs.}$$

(2) If no.s of obs. = n (even),

$$\text{Median} = \frac{1}{2}\left(\frac{n}{2}^{th}\text{obs.} + \left(\frac{n}{2}+1\right)^{th}\text{obs.}\right)$$

(3) For discrete series, first find cumulative frequencies & then apply rule (1) or (2);

(4) For grouped data,
first find cumulative frequency & then apply,

$$\boxed{\text{Median} = L + \frac{i}{f_0}\left(\frac{N}{2} - C\right)}$$

C → Cumulative freq. of pre-median class.

Where, L ⇒ Lower limit of median class.
i = Class size
f_0 = Freq. of median class.

N → Total No.s of observations
C → Cumulative freq. of ...

ARITHMETIC MEAN = BEST AVERAGE

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Q-1 Find Median for first fifty natural numbers

(a) 25
(b) ~~25.5~~
(c) 26
(d) None

Note:→ Here series is 1, 2, 3, ... 50.

∴ $n = 50$ (even)

∴ Median $= \frac{1}{2}(25^{th}\text{ obs.} + 26^{th}\text{ obs.})$

$\Rightarrow \frac{1}{2}(25+26)$

$\Rightarrow 25.5$

M.T. ⇒ For 1^{st} N natural Numbers;

Median = A.M. $= \frac{n+1}{2}$.

Q-2 Find Median for 2, 5, 7, −2, −3, 0, −2.

−3, −2, −2, 0, 2, 5, 7, 40, 70, 80

Inc. order

−3
−2
−2
0
2
5
7
40
70
80

Median $\Rightarrow \frac{2+5}{2} \Rightarrow 3.5$

Steps to follow:-

→ find cumulative frequency.
→ find Median class by finding half of Σf. e.g. ⇒ $\Sigma f = 75$ then $\frac{75+1}{2} \Rightarrow 38$ → look for this in c.f.

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Q-> Find Median Marks....

Marks Obtained	No. of Students (f)	cf
10	2	2
20	5	7
30	15	22
40	20	42
50	03	45
60	30	75
	$\Sigma f = 75$	

$\Rightarrow \left(\frac{75+1}{2}\right)^{th}$ term

Median $\Rightarrow$ 40

For Grouped Data

Cumulative frequency

Q->

Marks	No. of Studs	cf
0-10	2	2
10-20	5	7
20-30	15	22
30-40	20	42
40-50	3	45
50-60	30	75
	$\Sigma f = 75$	

$$\text{Median} = L + \frac{i}{f_0}\left(\frac{N}{2} - C\right)$$

Answer

Here N = 75

$\therefore \frac{N}{2} = 37.5$ & it decides Median class

$\therefore$ 30-40

$\therefore$ Median $= 30 + \frac{10}{20}\left(\frac{75}{2} - 22\right)$

$\Rightarrow 30 + \frac{1}{2}(15.5)$

$\Rightarrow 30 + 7.78$

Median = 37.75

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KARL PEARSON RELATION:-

Mode = 3 (Median) − 2 (Mean)
↓
A.M.

Note:-> (1) If mode = median = mean.
It is called Symmetrical distribution.

(2) If mode ≠ median ≠ mean.
It is called skewed distribution.

Example → If 10 students get same mark of 80,
then Mode = Mean = Median = 80.

Some Other formula:-

(1) Mean Deviation (M.D.)

$$M.D. = \frac{\Sigma(x-\bar{x})}{n}.$$

(2) Standard Deviation (S.D.)

$$SD = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}.$$

(3) VARIANCE = $(S.D.)^2$.

(4) Coefficient of S.D. = $\frac{S.D.}{Mean}$.

(5) Coefficient of ~~VARIANCE~~ VARIATION = $\frac{SD}{Mean} \times 100$.

Q.→ For first 5 natural numbers.
find (i) M.D.
(ii) S.D.
(iii) VARIANCE.

Solⁿ →

x	$\bar{x}$	$\lvert x-\bar{x}\rvert$	$\lvert x-\bar{x}\rvert^2$
1		2	4
2		1	1
3	2	0	0
4		1	1
5		2	4
		Sum ⇒ 6	Sum ⇒ 10.

→ M.D.

$\Rightarrow \frac{6}{5} \Rightarrow 1.2$

$$S.D. = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$$

$$\Rightarrow \sqrt{\frac{10}{5}}$$

$$\Rightarrow \sqrt{2}.$$

Variance $= 2$.

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M.I. ⇒ For first 'n' natural no.s,

① S.D. = $\sqrt{\frac{n^2-1}{12}}$

Variance = $\left(\frac{n^2-1}{12}\right)$.

} when n is even/odd.

~~M.D. = $\frac{\Sigma |x_i - \bar{x}|}{n}$~~

~~⇒ $\frac{6}{5}$ ⇒ 1.2~~

② If n is odd,

M.D. = $\frac{2}{n}\left[1+2+3+\ldots+\left(\frac{n-1}{2}\right)\right]$

~~Q.~~ For first eleven natural no.
Find M.D, S.D. & variance.

(i) S.D. ⇒ $\sqrt{10}$

(ii) V ⇒ 10

(iii) M.D. ⇒ $\frac{2}{11}\left[(1+2+3+4+5)\right]$

⇒ $\frac{2}{11}(15)$

⇒ $\frac{30}{11}$.

Q-> If V of a series is 64.

S.D. => 8.

Imp. Q-> Who is the best player among A, B & C.

Players	Mean $\bar{x}$	S.D.	Coeff of var. $\frac{S.D}{Mean} \times 100$
A	50	2	→ 4%
B	60	3	→ 5%
C	80	5	→ 6.25%

→ A is the best player.

RULE → Less the value of coefficient of variation, better is the player.

STATISTICAL DIAGRAMS

(1) HISTOGRAM:-

(i) It is graphical representation of grouped data.

(ii) It consists of number of rectangles.

Each rectangle has width equal to class width.

" " " height proportional to its frequency

Imp. ⇒ ∴ Area of rectangle is proportional to the frequency

V.V. Imp. (iii) Mode is determined using Histogram.

(iv) Rough Sketch